

Student Learning Map

Unit 3

Topic: Geometry Gallery

Course: Mathematics 1

Key Learning(s):

1. The interior and exterior angles of a polygon can be determined by the number of sides of the polygon.
2. There are special inequalities that exist between the sides of a triangle, the angles of a triangle, and between angles and their opposite sides.
3. Congruency of triangles can be determined by special relationships of the sides and angles.
4. Triangles have four different points of concurrency or points of center.
5. Special quadrilaterals are related by their properties.

Unit Essential Question(s):

In what real life situations would it be necessary to determine the interior or exterior angles of a polygon?
 How could the points of concurrency be used to solve a real life situation?
 What relationships between sides and angles can be used to prove the congruency of triangles?
 How are the parallelogram, rectangle, rhombus, square, trapezoid, and kite alike and different?

Optional Instructional Tools:
 Mira, patty paper, Geometer's Sketchpad, compass, straight edge, protractor, graphing calculator

Concept:

Measures of interior and exterior angles of polygons

Concept:

Triangular Inequalities

Lesson Essential Questions

1. What does the sum of two sides of a triangle tell me about the third side?
2. In a triangle, what is the relationship of an angle of a triangle and its opposite side?
3. What is the relationship between the exterior angle of a triangle and either of its remote interior angles?
4. For two triangles with two pairs of congruent corresponding sides, what is the relationship between the included angle and the third side?
5. How do I use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle?

Vocabulary

1. triangle inequality
2. side-angle inequality
3. side-side-side inequality
4. exterior angle inequality

Lesson Essential Questions

1. How do I find the sum of the measures of the interior angles of a polygon?
2. How do I find the measure of an interior angle of a regular polygon?
3. How do I find the sum of the measures of the exterior angles of a polygon and the measure of an exterior angle of a regular polygon?
4. What is the relationship between the exterior angle of a triangle and its 2 remote interior angles?

Vocabulary

1. convex polygon
2. concave polygon
3. remote interior angles

Concept:

Triangle Points of Concurrency

Lesson Essential Questions

1. How do I find points of concurrency in triangles?
2. How can I use points of concurrency in triangles?

Vocabulary

1. incenter
2. orthocenter
3. circumcenter
4. centroid

Concept:

Congruency Theorems for Triangles

Lesson Essential Questions

1. How can you justify that two triangles are congruent?

Vocabulary

1. SSS
2. SAS
3. ASA
4. AAS
5. LL
6. HA
7. LA
8. HL

Concept:

Relationships Between Special Quadrilaterals

Lesson Essential Questions

1. What characteristics differentiate the following quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite?
2. If 3 parallel lines cut off equal segments of one transversal, how would lengths of segments of any transversal be related?

Vocabulary:

1. Kite

Notes:

Vocabulary to Maintain
interior angle, exterior
angle, diagonal of a
polygon, regular
polygon

Notes:

Side-side-side inequality
is the converse of the
Hinge Theorem

Notes:

Notes:

Reference –corresponding
parts of congruent triangles
are congruent (CPCTC)

Notes:

Vocabulary to be
maintained:

1. quadrilateral
2. parallelogram
3. rectangle
4. rhombus
5. square
6. trapezoid
7. diagonal

Centers of Triangles Learning Task**Notes:**

This task provides a guided discovery and investigation of the points of concurrency in triangles. Students will construct and use the following points:

- *incenter*
- *orthocenter*
- *circumcenter*
- *centroid*

Supplies Needed:

- *miras (or a similar reflective tool)*
- *patty paper*
- *compasses*
- *straightedges*
- *Geometer's Sketchpad or similar geometry software, if possible*

Comments:

In this task students will determine the location for an amusement park by finding the centers of a triangle. The centers will be new to the students but not the constructions. Make sure students remember the significance of points on the perpendicular bisector of a segment (equidistant from the endpoints of the segment) and the points on an angle bisector (equidistant from the sides of the angle).

As students work through the tasks and present their solutions to the class make sure they emphasize the name of the center found, how it was found, and its significance.

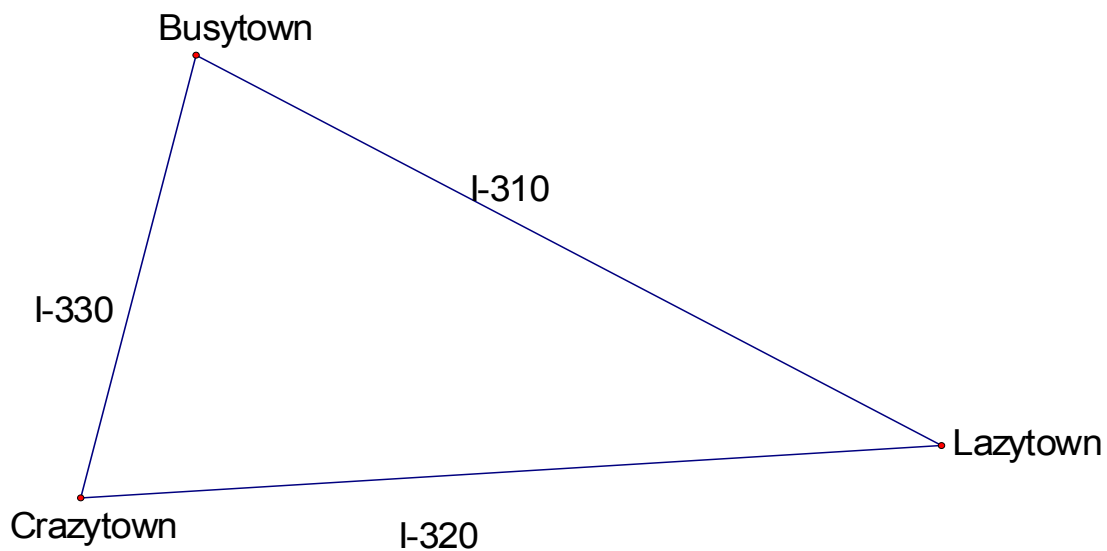
The significance of the circumcenter and incenter can be determined through measurement. If needed, encourage students to measure the distances from the triangle centers to the sides and vertices of the triangle. Students can use properties of the perpendicular bisectors and angle bisectors to justify their conjectures about the significance of the circumcenter and the incenter. Students may require help in determining the significance of the centroid (center of gravity). Students can determine one of the significant features of the centroid through measurement (the centroid is twice the distance from the vertex to the opposite side). Other than being the point of concurrency of the altitudes the orthocenter has no additional significance in this task.

<i>Triangle Center:</i>	<i>Point of Concurrency of:</i>	<i>Significance of:</i>
<i>Incenter</i>	<i>Angle bisectors</i>	<i>Center of inscribed circle Equidistant from the sides of the triangle</i>
<i>Circumcenter</i>	<i>Perpendicular bisectors</i>	<i>Center of the circumscribing circle</i>

		Equidistant from the vertices of the
<i>Orthocenter</i>	<i>Altitudes</i>	-
<i>Centroid</i>	<i>Medians</i>	<i>Center of balance or gravity</i> <i>The distance from a vertex to the centroid is half the distance from the centroid to the opposite side.</i>

Centers of Triangles Learning Task

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



1. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

Comments:

“Just by looking” is important here. Students need to take a moment to look at the towns and make a decision.

Solution:

Answers will vary.

2. Now you will use some mathematical concepts to help you choose a location for the tower.

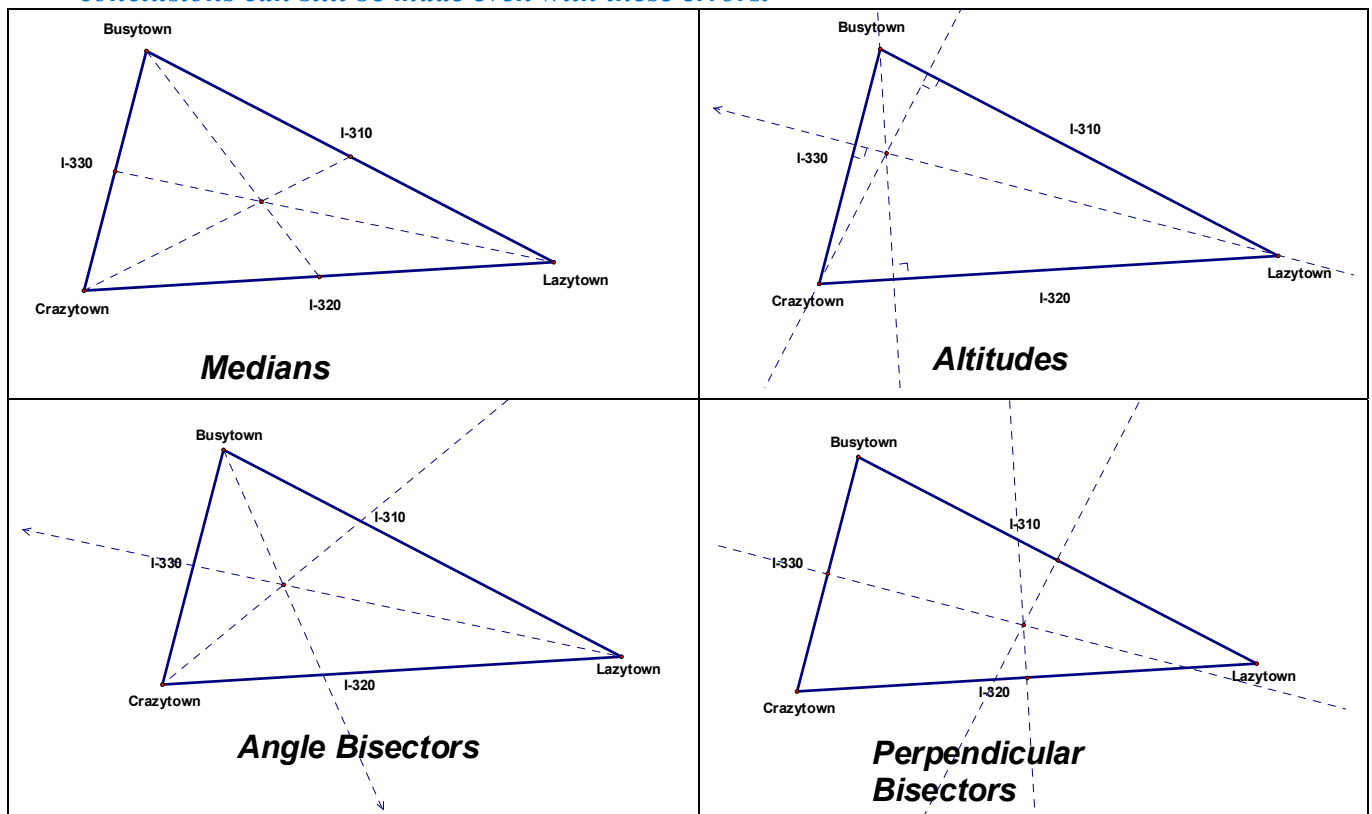
In the previous lesson, you learned how to construct medians and altitudes of triangles. In 7th grade, you learned how to construct angle bisectors and perpendicular bisectors. Investigate the problem above by constructing the following:

- all 3 medians of the triangle
- all 3 altitudes of the triangle
- all 3 angle bisectors of the triangle
- all 3 perpendicular bisectors of the triangle

You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

Not drawn to scale:

The constructions, regardless of which tool is used, should result in the following. It is very important for students to realize the three lines always intersect in a single point. Because some of the tools they are using are not very precise the students will have some errors in measurement. These errors should be discussed as a group but some conclusions can still be made even with these errors.



- Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.

Solution:

Answers will vary but it is critical for students to have a mathematical justification for their decision. For example, they may choose the circumcenter because it is equidistant

from all three cities. Or they may choose the incenter because it is equidistant from each of the roads. They could choose the centroid instead of the circumcenter because it is closer to two of the cities while not being that much further away from Lazytown.

4. How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?

Solution:

Answers will vary.

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the **points of concurrency** of a triangle.

The intersection point of the medians is called the **centroid** of the triangle.

The intersection point of the angle bisectors is called the **incenter** of the triangle.

The intersection point of the perpendicular bisectors is called the **circumcenter** of the triangle.

The intersection point of the altitudes is called the **orthocenter** of the triangle.

Comments:

Students struggle with using the terms point of concurrency and concurrent lines correctly. Make sure they understand what they mean and how to use them. To help them understand the significance of a point of concurrency, ask them to draw three lines on their paper, without looking at anyone else's paper. Then, ask whose lines are drawn in such a way that all three intersect at the same point. Have them compare their drawing and determine the different way three lines can be related to each other.

5. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

Comments:

Students will need to have an idea of the significance of the triangle centers in order to answer this question. If they have not already done so, they need to go back to their constructions and explore the properties of the triangle centers they found in #2. Some groups may have discovered these properties already.

Make sure the name of the center, how it was found, and its significance are emphasized as students present their solutions.

Solution:

Answers will vary.

6. Which triangle center did you recommend for the location of the amusement park?

Solution:

The students are only being asked to name their point. They need to decide if their chosen point is the centroid, incenter, circumcenter, or orthocenter based on the definitions above.

7. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.

Comments:

Since the president is concerned about the cost of the roads, students need to take that into account in their memo. In consideration of the cost of building the roads, some groups may want to change their earlier decision. For example, if a group chose to use the circumcenter because it was an equal distance from each of the cities they may want to choose the incenter or median to reduce the cost of building the roads. Or, students could measure the total distance from a point of concurrency to each of the cities and choose the center that gives the shortest total distance.

Solution:

Answers may vary. Mathematical justification of the answer is the most important aspect of this activity.

Location Learning Tasks

Comments:

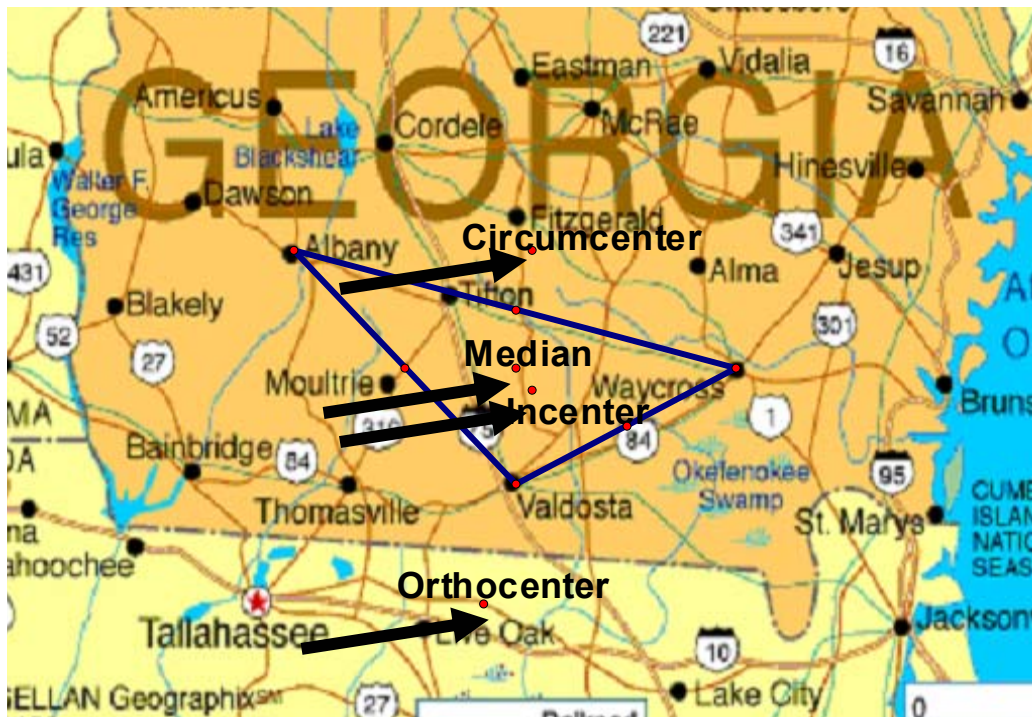
These tasks present an opportunity for students to apply the knowledge gained in the Amusement Park Task to two new scenarios. These are very open ended tasks and could be assessed using a rubric valuing the mathematical content, reasoning, and communication skills demonstrated by students in their solutions. A rubric should be based on the Content Standard MM1G3e and the Process Standards, MM1P1 through MM1P5.

Cell Tower

A cell service operator plans to build an additional tower so that more of the southern part of Georgia has stronger service. People have complained that they are losing service, so the operator wants to remedy the situation before they lose customers. The service provider looked at the map of Georgia below and decided that the three cities: Albany, Valdosta, or Waycross, were good candidates for the tower. However, some of the planners argued that the cell tower would provide a more powerful signal within the entire area if it were placed somewhere between those three cities. Help the service operator decide on the best location for the cell tower.

Solutions:

The triangle centers are indicated below. The students may choose different points as the solution to the problem. They must be able to justify mathematically why they chose that point. For example, they might choose the circumcenter because it is equidistant from the three cities. They might choose the incenter because it is equidistant from the three highways. Or, they might choose the median because it is closer to being in the center of the three cities.



Compose a memo to the president of the cell company justifying your final choice for the location of the tower. Use appropriate mathematical vocabulary and reasoning in your justification.

Solution:

Answers will vary. Students should address their mathematical reasoning behind their choice. See comments above about using a rubric for grading purposes.

Burn Center

Now, you can apply your new found knowledge to a more important situation:

Because of the possible closing of Grady Hospital, several groups have decided to fund an acute trauma hospital located where it would give the most people of Georgia the most access. Using one of the sets of cities identified below, or your choices of three cities, and the map from the earlier question, decide where the new trauma hospital should be located. Be sure to mathematically justify your answer.

City set one: Bainbridge, Savannah, and Dalton

City set two: Thomasville, Sylvania, and Calhoun

City set three: Valdosta, Augusta, and Marietta

Solution: